Effect of spin-conserving scattering on Gilbert damping in ferromagnetic semiconductors

K. Shen,[1] G. Tatara,[2] and M. W. Wu[1,*]
[1]*Hefei National Laboratory for Physical Sciences at Microscale and Department of Physics, University of Science and Technology of China, Hefei, Anhui 230026, China*
[2]*Department of Physics, Tokyo Metropolitan University, Hachioji, Tokyo 192-0397, Japan*
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The Gilbert damping in ferromagnetic semiconductors is theoretically investigated based on the *s-d* model. In contrast to the situation in metals, all the spin-conserving scattering in ferromagnetic semiconductors supplies an additional spin-relaxation channel due to the momentum-dependent effective magnetic field of the spin-orbit coupling, thereby modifies the Gilbert damping. In the presence of a pure spin current, we predict a contribution due to the interplay of the anisotropic spin-orbit coupling and a pure spin current.

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The ferromagnetic systems have attracted much attention both for the abundant fundamental physics and promising applications in the past decade.[1,2] The study on the collective magnetization dynamics in such systems has been an active field with the aim to control the magnetization. In the literature, the magnetization dynamics is usually described by the phenomenological Landau-Lifshitz-Gilbert equation,[3]

$$\dot{\mathbf{n}} = \gamma \mathbf{H}_{\text{eff}} \times \mathbf{n} + \alpha \mathbf{n} \times \dot{\mathbf{n}} \tag{1}$$

with $\mathbf{n}$ denoting the direction of the magnetization. The first and second terms on the right-hand side of the equation represent the precession and relaxation of the magnetization under the effective magnetic field $\mathbf{H}_{\text{eff}}$, respectively. The relaxation term is conventionally named as the Gilbert-damping term with the damping coefficient α. The time scale of the magnetization relaxation then can be estimated by $1/(\alpha\gamma H_{\text{eff}})$,[4] which is an important parameter for dynamic manipulations. The coefficient α is essential in determining the efficiency of the current-induced magnetization switching, and experimental determination of α has been carried out intensively in metals[5] and magnetic semiconductors.[6]

To date, many efforts have been made to clarify the microscopic origin of the Gilbert damping.[7–12] Kohno *et al.*[8] employed the standard diagrammatic perturbation approach to calculate the spin torque in the small-amplitude magnetization dynamics and obtained a Gilbert torque with the damping coefficient inversely proportional to the electron-spin lifetime. They showed that the electron-nonmagnetic impurity scattering, a spin-conserving process, does not affect the Gilbert damping. Later, they extended the theory into the finite-amplitude dynamics by introducing an SU(2) gauge field[2] and obtained a Gilbert torque identical to that in the case of small-amplitude dynamics.[9] In those calculations, the electron-phonon and electron-electron scatterings were discarded. One may infer that both of them should be irrelevant to the Gilbert damping in ferromagnetic metals since they are independent of the electron-spin relaxation somewhat like the electron-nonmagnetic impurity scattering. However, the situation is quite different in ferromagnetic semiconductors, where the spin-orbit coupling (SOC) due to the bulk inversion asymmetry[13] and/or the structure inversion asymmetry[14] presents a momentum-dependent effective magnetic field (inhomogeneous broadening[15]). As a result, any spin-conserving scattering, including the electron-electron Coulomb scattering, can randomize the spin precession of the itinerant electron and hence results in a spin-relaxation channel [i.e., the D'yakonov-Perel' (DP) mechanism[16]] which affects the Gilbert damping. In this case, many-body effects on the Gilbert damping due to the electron-electron Coulomb scattering should be expected. Sinova *et al.*[17] studied the Gilbert damping in GaMnAs ferromagnetic semiconductors by including the SOC to the energy-band structure. In that work, the dynamics of the carrier spin coherence was missed.[18] The issue of the present work is to study the Gilbert damping in a coherent frame.

In this Brief Report, we apply the gauge-field approach to investigate the Gilbert damping in ferromagnetic semiconductors. In our frame, all the relevant scattering processes, even the electron-electron scattering which gives rise to many-body effects, can be included. The goal of this work is to illustrate the role of the SOC and spin-conserving scattering on Gilbert damping. We show that the spin-conserving scattering can affect the Gilbert damping due to the contribution on spin-relaxation process. We also discuss the case with a pure spin current, from which we predict a Gilbert torque due to the interplay of the SOC and the spin current.

Our calculation is based on the *s-d* model with itinerant *s* and localized *d* electrons. The collective magnetization arising from the *d* electrons is denoted by $\mathbf{M}=M_s\mathbf{n}$. The exchange interaction between itinerant and localized electrons can be written as $H_{\text{sd}}=M\int d\mathbf{r}(\mathbf{n}\cdot\boldsymbol{\sigma})$, where the Pauli matrices $\boldsymbol{\sigma}$ are spin operators of the itinerant electrons and M is the coupling constant. In order to treat the magnetization dynamics with an arbitrary amplitude,[9] we define the temporal spinor operators of the itinerant electrons $a(t)=[a_\uparrow(t),a_\downarrow(t)]^T$ in the rotation coordinate system with $\uparrow(\downarrow)$ labeling the spin orientation parallel (antiparallel) to $\mathbf{n}$. With a unitary transformation matrix $U(t)$, one can connect the operators $a_{\uparrow(\downarrow)}$ with those defined in the lattice coordinate system $c_{\uparrow(\downarrow)}$ by $a(t)=U(t)c$. Then, an SU(2) gauge field $A_\mu(t)=-iU(t)^\dagger[\partial_\mu U(t)]=\mathbf{A}_\mu(t)\cdot\boldsymbol{\sigma}$ should be introduced into the rotation framework to guarantee the invariance of the total Lagrangian.[9] In the slow and smooth precession limit, the gauge field can be treated perturbatively.[9] Besides, one needs a time-dependent 3×3 orthogonal rotation matrix $\mathcal{R}(t)$, which obeys $U^\dagger\boldsymbol{\sigma}U=\mathcal{R}\boldsymbol{\sigma}$, to transform any vector between the two coordinate systems. More details can be found in Ref. 2. In the following, we restrict our derivation to a spatially homogeneous system, to obtain the Gilbert-damping torque.

Up to the first order, the interaction Hamiltonian due to the gauge field is $H_A=\Sigma_{\mathbf{k}}\mathbf{A}_0\cdot a_{\mathbf{k}}^{\dagger}\boldsymbol{\sigma}a_{\mathbf{k}}$ and the spin-orbit coupling reads

$$H_{so}=\frac{1}{2}\sum_{\mathbf{k}}\mathbf{h}_{\mathbf{k}}\cdot c^{\dagger}\boldsymbol{\sigma}c=\frac{1}{2}\sum_{\mathbf{k}}\tilde{\mathbf{h}}_{\mathbf{k}}\cdot a_{\mathbf{k}}^{\dagger}\boldsymbol{\sigma}a_{\mathbf{k}} \quad (2)$$

with $\tilde{\mathbf{h}}_{\mathbf{k}}=\mathcal{R}\mathbf{h}_{\mathbf{k}}$ representing the momentum-dependent effective magnetic field.[13,14] Here, we take the Planck constant $\hbar=1$. We start from the fully microscopic kinetic spin-Bloch equations of the itinerant electrons derived from the nonequilibrium Green's-function approach,[15,19]

$$\partial_t\rho_{\mathbf{k}}=\partial_t\rho_{\mathbf{k}}|_{\text{coh}}+\partial_t\rho_{\mathbf{k}}|_{\text{scat}}^{c}+\partial_t\rho_{\mathbf{k}}|_{\text{scat}}^{f}, \quad (3)$$

where $\rho_{\mathbf{k}}$ represent the itinerant electron-density matrices defined in the rotation coordinate system. The coherent term can be written as

$$\partial_t\rho_{\mathbf{k}}|_{\text{coh}}=-i[\mathcal{A}\cdot\boldsymbol{\sigma},\rho_{\mathbf{k}}]-i\left[\frac{1}{2}\tilde{\mathbf{h}}_{\mathbf{k}}\cdot\boldsymbol{\sigma}+\hat{\Sigma}_{\text{HF}},\rho_{\mathbf{k}}\right]. \quad (4)$$

Here [,] is the commutator and $\mathcal{A}(t)=\mathbf{A}_0(t)+M\hat{\mathbf{z}}$ with $\mathbf{A}_0$ and $M\hat{\mathbf{z}}$ representing the gauge field and effective magnetic filed due to s-d exchange interaction, respectively. $\hat{\Sigma}_{\text{HF}}$ is the Coulomb Hartree-Fock term of the electron-electron interaction. $\partial_t\rho_{\mathbf{k}}|_{\text{scat}}^{c}$ and $\partial_t\rho_{\mathbf{k}}|_{\text{scat}}^{f}$ in Eq. (3) include all the relevant spin-conserving and spin-flip scattering processes, respectively.

The spin-flip term $\partial_t\rho_{\mathbf{k}}|_{\text{scat}}^{f}$ results in the damping effect was studied in Ref. 9. Let us confirm this by considering the case of the magnetic disorder $V_{\text{imp}}^{m}=u_s\Sigma_j\tilde{\mathbf{S}}_j\cdot a^{\dagger}\boldsymbol{\sigma}a\,\delta(\mathbf{r}-\mathbf{R}_j)$. The spin-flip part then reads

$$\partial_t\rho_{\mathbf{k}}|_{\text{scat}}^{f}=\partial_t\rho_{\mathbf{k}}|_{\text{scat}}^{f(0)}+\partial_t\rho_{\mathbf{k}}|_{\text{scat}}^{f(1)} \quad (5)$$

with $\partial_t\rho_{\mathbf{k}}|_{\text{scat}}^{f(i)}$ standing for the ith-order term with respect to the gauge field, i.e.,

$$\partial_t\rho_{\mathbf{k}}|_{\text{scat}}^{f(0)}=-\frac{\pi n_s u_s^2 S_{\text{imp}}^2}{3}\sum_{\mathbf{k}_1\eta_1\eta_2}\sigma^{\alpha}\rho_{\mathbf{k}_1}^{>}(t)T_{\eta_1}\sigma^{\alpha}T_{\eta_2}\rho_{\mathbf{k}}^{<}(t)\times\delta(\epsilon_{\mathbf{k}_1\eta_1}-\epsilon_{\mathbf{k}\eta_2})-(>\leftrightarrow<)+\text{H.c.}, \quad (6)$$

$$\partial_t\rho_{\mathbf{k}}|_{\text{scat}}^{f(1)}=\frac{i2\pi n_s u_s^2 S_{\text{imp}}^2}{3}\varepsilon^{\alpha\beta\gamma}A_0^{\gamma}(t)\sum_{\mathbf{k}_1\eta_1\eta_2}\sigma^{\alpha}\rho_{\mathbf{k}_1}^{>}(t)T_{\eta_1}\sigma^{\beta}T_{\eta_2}\rho_{\mathbf{k}}^{<}(t)\times\frac{d}{d\epsilon_{\mathbf{k}_1\eta_1}}\delta(\epsilon_{\mathbf{k}_1\eta_1}-\epsilon_{\mathbf{k}\eta_2})-(>\leftrightarrow<)+\text{H.c.}, \quad (7)$$

where $T_{\eta}(i,j)=\delta^{\eta i}\delta^{\eta j}$ for the spin band η. Here $\rho_{\mathbf{k}}^{>}=1-\rho_{\mathbf{k}}$ and $\rho_{\mathbf{k}}^{<}=\rho_{\mathbf{k}}$. $(>\leftrightarrow<)$ is obtained by interchanging $>$ and $<$ from the first term in each equation. ε^{ijk} is the Levi-Civita permutation symbol. The gauge-field term, $\partial_t\rho_{\mathbf{k}}|_{\text{scat}}^{f(1)}$, results from the spin correlation of a single magnetic impurity at different times.[9] It induces a spin polarization proportional to $\hat{\mathbf{z}}\times\mathbf{A}_0^{\perp}(t)$ which gives a Gilbert torque. The damping coefficient is inversely proportional to the spin-relaxation time τ_s determined by the spin-flip scattering $\partial_t\rho_{\mathbf{k}}|_{\text{scat}}^{f(0)}$. The spin-flip scattering term in Eq. (3) thus reproduces the result of Ref. 9.

We now demonstrate that the Gilbert-damping torque arises also from the spin-conserving scattering. For the discussion of the spin-conserving term, it is sufficient to approximate the spin-flip term as $\partial_t\rho_{\mathbf{k}}|_{\text{scat}}^{f}=-(\rho_{\mathbf{k}}-\rho_{\mathbf{k}}^{e})/\tau_s$, with $\rho_{\mathbf{k}}^{e}$ representing the instantaneous equilibrium distribution (i.e., $\rho_{\mathbf{k}}^{e}$ is $\rho_{\mathbf{k}}$ without the gauge field and $\mathbf{P}_{\mathbf{k}}^{s}$). Equation (3) then reads

$$\partial_t\rho_{\mathbf{k}}=-i[\mathcal{A}\cdot\boldsymbol{\sigma},\rho_{\mathbf{k}}]-i\left[\frac{1}{2}\tilde{\mathbf{h}}_{\mathbf{k}}\cdot\boldsymbol{\sigma},\rho_{\mathbf{k}}\right]+\partial_t\rho_{\mathbf{k}}|_{\text{scat}}^{c}-(\rho_{\mathbf{k}}-\rho_{\mathbf{k}}^{e})/\tau_s+\mathbf{P}_{\mathbf{k}}^{s}. \quad (8)$$

Here, we add an additional term, $\mathbf{P}_{\mathbf{k}}^{s}$, to describe the source of a pure spin current due to the magnetization dynamic pumping[4] or electrically injection[20,21] in order to discuss the system with a pure spin current. We neglect the Coulomb Hartree-Fock effective magnetic field since it is approximately parallel to the s-d exchange field but with a smaller magnitude.

By averaging density matrices over the momentum direction, one obtains the isotropic component $\rho_{i,k}=\int\frac{d\Omega_{\mathbf{k}}}{4\pi}\rho_{\mathbf{k}}$. The anisotropic component is then expressed as $\rho_{a,\mathbf{k}}=\rho_{\mathbf{k}}-\rho_{i,k}$. It is obvious that this anisotropic component does not give any spin torque in the absence of the SOC since $\Sigma_{\mathbf{k}}\text{Tr}(\boldsymbol{\sigma}\rho_{a,\mathbf{k}})=0$. Below, it is shown that this component leads to the damping when coupled to the spin-orbit interaction.

By denoting the isotropic component of the equilibrium part $(\rho_{\mathbf{k}}^{e})$ as $\rho_{i,k}^{e}$ and representing the nonequilibrium isotropic part by $\delta\rho_{i,k}=\rho_{i,k}-\rho_{i,k}^{e}$, we write the kinetic spin-Bloch equations of the nonequilibrium isotropic density matrices $\delta\rho_{i,k}$ and those of the anisotropic components $\rho_{a,\mathbf{k}}$ as

$$\partial_t\rho_{i,k}=-\frac{\delta\rho_{i,k}}{\tau_s}-i[\mathcal{A}\cdot\boldsymbol{\sigma},\delta\rho_{i,k}]-i\overline{\left[\frac{1}{2}\tilde{\mathbf{h}}_{\mathbf{k}}\cdot\boldsymbol{\sigma},\rho_{a,\mathbf{k}}\right]}-i[\mathbf{A}_0\cdot\boldsymbol{\sigma},\rho_{i,k}^{e}], \quad (9)$$

$$\partial_t\rho_{a,\mathbf{k}}=\partial_t\rho_{a,\mathbf{k}}|_{\text{scat}}^{c}-i[\mathcal{A}\cdot\boldsymbol{\sigma},\rho_{a,\mathbf{k}}]-i\left[\frac{1}{2}\tilde{\mathbf{h}}_{\mathbf{k}}\cdot\boldsymbol{\sigma},\delta\rho_{i,k}\right]-i\left[\frac{1}{2}\tilde{\mathbf{h}}_{\mathbf{k}}\cdot\boldsymbol{\sigma},\rho_{a,\mathbf{k}}\right]+i\overline{\left[\frac{1}{2}\tilde{\mathbf{h}}_{\mathbf{k}}\cdot\boldsymbol{\sigma},\rho_{a,\mathbf{k}}\right]}+\mathbf{P}_{\mathbf{k}}^{s}, \quad (10)$$

respectively. The overline in these equations presents an angular average over the momentum space.

We further define $\rho_{a,\mathbf{k}}^{(0)}$ as the anisotropic density in the absence of the gauge field, $\mathbf{A}_0$. As easily seen, it vanishes when $\mathbf{P}_{\mathbf{k}}^{s}=0$. The anisotropic component involving the gauge field is denoted by $\rho_{a,\mathbf{k}}^{(1)}=\rho_{a,\mathbf{k}}-\rho_{a,\mathbf{k}}^{(0)}$. Equation (10) is expressed in terms of these components as

$$\partial_t\rho_{a,\mathbf{k}}^{(0)}=-i[\mathbf{M}\cdot\boldsymbol{\sigma},\rho_{a,\mathbf{k}}^{(0)}]+\partial_t\rho_{a,\mathbf{k}}^{(0)}|_{\text{scat}}^{c}+\mathbf{P}_{\mathbf{k}}^{s}-i\left[\frac{1}{2}\tilde{\mathbf{h}}_{\mathbf{k}}\cdot\boldsymbol{\sigma},\rho_{a,\mathbf{k}}^{(0)}\right]+i\overline{\left[\frac{1}{2}\tilde{\mathbf{h}}_{\mathbf{k}}\cdot\boldsymbol{\sigma},\rho_{a,\mathbf{k}}^{(0)}\right]}, \quad (11)$$

$$\partial_t\rho^{(1)}_{a,\mathbf{k}} = \partial_t\rho^{(1)}_{a,\mathbf{k}}|^c_{\text{scat}} - i[\mathcal{A}\cdot\boldsymbol{\sigma},\rho^{(1)}_{a,\mathbf{k}}] - i\left[\frac{1}{2}\tilde{\mathbf{h}}_{\mathbf{k}}\cdot\boldsymbol{\sigma},\delta\rho_{i,k}\right] - i\left[\frac{1}{2}\tilde{\mathbf{h}}_{\mathbf{k}}\cdot\boldsymbol{\sigma},\rho^{(1)}_{a,\mathbf{k}}\right] - i[\mathbf{A}_0\cdot\boldsymbol{\sigma},\rho^{(0)}_{a,\mathbf{k}}]. \quad (12)$$

Within the elastic-scattering approximation, the electron-phonon scattering as well as the electron-nonmagnetic impurity scattering can be simply written as $\Sigma_{l,m}\rho^{(1)}_{a,k,lm}Y_{lm}/\tau_l$, where the density matrices are expanded by the spherical harmonics functions Y_{lm}, i.e., $\rho^{(1)}_{a,k,lm}=\int\frac{d\Omega_{\mathbf{k}}}{4\pi}\rho^{(1)}_{a,\mathbf{k}}Y_{lm}$. τ_l is the effective momentum relaxation time. The exact calculation of the Coulomb scattering is more complicated. Nevertheless, one can still express this term in the form of $\rho^{(1)}_{a,\mathbf{k}}/F_{\mathbf{k}}(\rho)$, where $F_{\mathbf{k}}$ is a function of the density matrices[22] and reflects many-body effects. For simplification, we just introduce a uniform momentum relaxation time τ^*_l in the following calculation. Expanding Eq. (12) by the spherical harmonics functions, one obtains

$$\partial_t\rho^{(1)}_{a,k,lm} = - i[\mathcal{A}\cdot\boldsymbol{\sigma},\rho^{(1)}_{a,k,lm}] - i\left[\frac{1}{2}\tilde{\mathbf{h}}_{k,lm}\cdot\boldsymbol{\sigma},\delta\rho_{i,k}\right] - i[\mathbf{A}_0\cdot\boldsymbol{\sigma},\rho^{(0)}_{a,k,lm}] - i\left[\frac{1}{2}\tilde{\mathbf{h}}_{\mathbf{k}}\cdot\boldsymbol{\sigma},\rho^{(1)}_{a,\mathbf{k}}\right]_{lm} - \frac{\rho^{(1)}_{a,k,lm}}{\tau^*_l}, \quad (13)$$

where the expansion coefficient of any term $f_{\mathbf{k}}$ is expressed as $f_{k,lm}=\int\frac{d\Omega_{\mathbf{k}}}{4\pi}f_{\mathbf{k}}Y_{lm}$. In the strong scattering regime, i.e., $\frac{1}{\tau^*_l} \gg M$ and $\frac{1}{\tau^*_l}\gg|\mathbf{h}_{\mathbf{k}}|$, the first and fourth terms are much smaller than the last term, hence can be discarded from the right side. By taking the fact that the time derivative is a higher-order term into account, one also neglects $\partial_t\rho^{(1)}_{a,k,lm}$. The solution of Eq. (13) can be written as

$$\rho^{(1)}_{a,k,lm} = - i\tau^*_l\left\{\left[\frac{1}{2}\tilde{\mathbf{h}}_{k,lm}\cdot\boldsymbol{\sigma},\delta\rho_{i,k}\right] + [\mathbf{A}_0\cdot\boldsymbol{\sigma},\rho^{(0)}_{a,k,lm}]\right\}. \quad (14)$$

Substituting it into Eq. (14) and rewriting the equation in the leading order, one obtains

$$\partial_t\rho_{i,k} = - i(\mathcal{A}\cdot\boldsymbol{\sigma},\delta\rho_{i,k}) - \frac{i}{2}\overline{[\tilde{\mathbf{h}}_{\mathbf{k}}\cdot\boldsymbol{\sigma},\rho^{(0)}_{a,\mathbf{k}}]} - i(\mathbf{A}_0\cdot\boldsymbol{\sigma},\rho^e_{i,k}) - \sum_{lm}\frac{\tau^*_l}{4}[\tilde{\mathbf{h}}_{k,lm}\cdot\boldsymbol{\sigma},(\tilde{\mathbf{h}}_{k,lm}\cdot\boldsymbol{\sigma},\delta\rho_{i,k})] - \frac{\delta\rho_{i,k}}{\tau_s}. \quad (15)$$

The fourth term on the right-hand side of the equation is proportional to the second-order term of the SOC, which gives the spin-dephasing channel due to the DP mechanism.[16] This term can be expressed by $\tau^{-1}_{\text{DP}}\delta\rho_{i,k}$ with τ^{-1}_{DP} standing for the spin dephasing rate tensor, which can be written as $(\tau^{-1}_{\text{DP}})_{i,j}=\Sigma_{l,m}\langle\tau^*_l[(\mathbf{h}_{k,lm})^2\delta_{ij}-h^i_{k,lm}h^j_{k,lm}]\rangle$ by performing the ensemble averaging over the electron distribution. In the following, we treat τ_{DP} as a scalar for simplification and define the total spin lifetime as

$$\tau_r = 1/(\tau^{-1}_{\text{DP}} + \tau^{-1}_s), \quad (16)$$

then the combination of the last two terms in Eq. (15) gives $\delta\rho_{i,k}/\tau_r$. Similar to the previous procedure, we discard $\partial_t\rho_{i,k}$ in Eq. (15) and obtain

$$i[\mathcal{A}\cdot\boldsymbol{\sigma},\delta\rho_{i,k}] + \delta\rho_{i,k}/\tau_r = - i\overline{\left[\frac{1}{2}\tilde{\mathbf{h}}_{\mathbf{k}}\cdot\boldsymbol{\sigma},\rho^{(0)}_{a,\mathbf{k}}\right]} - i[\mathbf{A}_0\cdot\boldsymbol{\sigma},\rho^e_{i,k}]. \quad (17)$$

By taking $\delta\tilde{\mathbf{s}}_i=\frac{1}{2}\Sigma_{\mathbf{k}}\text{Tr}(\boldsymbol{\sigma}\delta\rho_{i,k})$, $\tilde{\mathbf{s}}^e_i=\frac{1}{2}\Sigma_{\mathbf{k}}\text{Tr}(\boldsymbol{\sigma}\rho^e_{i,k})$, and $\tilde{\mathbf{s}}^{(0)}_{a,\mathbf{k}}=\frac{1}{2}\text{Tr}(\boldsymbol{\sigma}\rho^{(0)}_{a,\mathbf{k}})$, one can write the solution as

$$\delta\tilde{\mathbf{s}}_i = \frac{\tilde{\mathbf{v}} + 2\tau_r\mathcal{A}\times\tilde{\mathbf{v}} + 4\tau^2_r(\tilde{\mathbf{v}}\cdot\mathcal{A})\mathcal{A}}{1 + 4|\mathcal{A}|^2\tau^2_r} - \tilde{\mathbf{s}}^e_i, \quad (18)$$

where $\tilde{\mathbf{v}}=\tilde{\mathbf{s}}^e_i+\tau_r\Sigma_{\mathbf{k}}\tilde{\mathbf{h}}_{\mathbf{k}}\times\tilde{\mathbf{s}}^{(0)}_{a,\mathbf{k}}$. $\tilde{\mathbf{s}}^e_i$ is just the equilibrium spin density, which is parallel to the magnetization, i.e., $\tilde{\mathbf{s}}^e_i=\tilde{s}^e_i\hat{\mathbf{z}}$. Now, we pick up the transverse component in the form of $\hat{\mathbf{z}}\times\mathbf{A}^{\perp}_0$, $\delta\tilde{\mathbf{s}}^{\perp}$, since only this component results in a Gilbert torque of the magnetization as mentioned above. We come to

$$\delta\tilde{\mathbf{s}}^{\perp} = 2\tilde{v}_z(\mathbf{A}^{\perp}_0\times\hat{\mathbf{z}})\tau^2_{\text{ex}}\tau_r/(\tau^2_r + \tau^2_{\text{ex}}) \quad (19)$$

with $\tau_{\text{ex}}=1/(2M)$. By transforming it back to the lattice coordinate system with $\mathcal{R}(\hat{\mathbf{z}}\times\mathbf{A}^{\perp}_0)=\frac{1}{2}\partial_t\mathbf{n}$,[9] one obtains

$$\delta\mathbf{s}^{\perp} = - \tilde{v}_z(\partial_t\mathbf{n})\tau^2_{\text{ex}}\tau_r/(\tau^2_r + \tau^2_{\text{ex}}). \quad (20)$$

This nonequilibrium spin polarization results in a spin torque performed on the magnetization according to $\mathbf{T}=-2M\mathbf{n}\times\delta\mathbf{s}$, i.e.,

$$\mathbf{T} = \tilde{v}_z(\mathbf{n}\times\partial_t\mathbf{n})\tau_{\text{ex}}\tau_r/(\tau^2_r + \tau^2_{\text{ex}}). \quad (21)$$

Compared with Eq. (1), the modification of the Gilbert-damping coefficient from this torque is

$$\alpha = \tilde{v}_z\tau_{\text{ex}}\tau_r/(M_s\tau^2_r + M_s\tau^2_{\text{ex}}), \quad (22)$$

We first discuss the case without the source term of the spin current. In this case, the anisotropic component $\rho^{(0)}_{a,\mathbf{k}}$ vanishes and $\tilde{v}_z=\tilde{s}^e_i$. We see that the Gilbert damping then arises from $1/\tau_r$ [Eq. (16)], i.e., from both the spin-flip scattering and the DP mechanism.[16] Our main message is that this DP contribution is affected by the spin-conserving scattering processes such as the electron-electron interaction and phonons. The temperature dependence of the Gilbert damping and the current-induced magnetization switching can thus be discussed quantitatively by evaluating τ_r. We note that our result reduces to the results of previous works[7,9] when only the spin-flip scattering is considered.

We should point out that our formalism applies also to metals, by considering the case $\frac{1}{\tau^*_l}\ll M$. In this case, the last term of Eq. (13) can be neglected and the effect of the spin-conserving scattering through τ^*_l becomes irrelevant.

When the pure spin current is included, we found additional contribution due to the interplay of the spin current and the SOC since we have

$$\tilde{v}_z = \tilde{s}_i^e + \left[\mathcal{R}\left(\tau_r \sum_{\mathbf{k}} \mathbf{h}_{\mathbf{k}} \times \mathbf{s}_{a,\mathbf{k}}^{(0)} \right) \right]_z = \tilde{s}_i^e + \tilde{s}_z^{\mathrm{sc}} \qquad (23)$$

with the spin current associated term $\tilde{s}_z^{\mathrm{sc}}$ defined accordingly. The origin of $\tilde{s}_z^{\mathrm{sc}}$ can be understood as follows. The anisotropic spin polarization $\mathbf{s}_{a,\mathbf{k}}^{(0)}$ arising from the pure spin current rotates around the SOC effective magnetic field $\mathbf{h}_{\mathbf{k}}$, which is also anisotropic. This precession finally results in an isotropic spin polarization $\mathbf{s}^{\mathrm{sc}} = \tau_r \Sigma_{\mathbf{k}} \mathbf{h}_{\mathbf{k}} \times \mathbf{s}_{a,\mathbf{k}}^{(0)}$ in the presence of spin relaxation. This term contributes to the spin polarization of the itinerant electrons along the direction of the magnetization, i.e., $\tilde{s}_z^{\mathrm{sc}}$, thereby modifies the Gilbert-damping term by $\tilde{s}_z^{\mathrm{sc}} / \tilde{s}_i^e$.

The additional Gilbert damping due to the spin current found here is different from the enhancement of the damping in the spin-pumping systems, where the existence of the interface is essential.[4] In other words, what contributes there is the divergence of the spin current, as is understood from the continuity equation for the spin, indicating that the spin damping is equal to $\nabla \cdot \mathbf{j}_s + \dot{s}$ (s is the total spin density). In contrast, the damping found in the present Brief Report arises even when the spin current is uniform if the spin-orbit interaction is there.

In summary, we have shown that the spin-conserving scatterings in ferromagnetic semiconductors, such as the electron-electron, electron-phonon, and electron-nonmagnetic impurity scatterings, contribute to the Gilbert damping in the presence of the SOC because of the inhomogeneous broadening effect. We also predict that a Gilbert torque arises from a pure spin current when coupled to the spin-orbit interaction.

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*Author to whom correspondence should be addressed; mwwu@ustc.edu.cn

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